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General $SU(2)_L \times SU(2)_R \times U(1)_{EM}$ Sigma Model With External Sources, Dynamical Breaking And Spontaneous Vacuum Symmetry Breaking

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Abstract

We give a general $SU(2)_L \times SU(2)_R \times U(1)_{EM}$ sigma model with external sources, dynamical breaking and spontaneous vacuum symmetry breaking, and present the general formulation of the model. It is found that σ and π^0 without electric charges have electromagnetic interaction effects coming from their internal structure. A general Lorentz transformation relative to external sources $J_{gauge} = (J_{A_\mu}, J_{A_\mu^\kappa})$ is derived, using the general Lorentz transformation and the four-dimensional current of nuclear matter of the ground state with $J_{gauge} = 0$, we give the four-dimensional general relations between the different currents of nuclear matter systems with $J_{gauge} \neq 0$ and those with $J_{gauge} = 0$. The relation of the density's coupling with external magnetic field is derived , which conforms well to dense nuclear matter in a strong magnetic field. We show different condensed effects in strong interaction about fermions and antifermions, and give the concrete scalar and pseudoscalar condensed expressions of σ_0 and π_0 bosons. About different dynamical breaking and spontaneous vacuum symmetry breaking, the concrete expressions of different mass spectra are obtained in field theory. This paper acquires the running spontaneous vacuum breaking value σ_0', and obtains the spontaneous vacuum breaking in terms of the running σ_0', which make nucleon, σ and π particles gain effective masses. We achieve both the effect of external sources and nonvanishing value of the condensed scalar and pseudoscalar paticles. It is deduced that the masses of nucleons, σ and π generally depend on different external sources.

1 Introduction

It is well known that a lot of unified models of the electroweak and the strong interactions utilize symmetry breaking. Now it is widely believed that the underlying laws of the world have essential

symmetries[1]. It is that symmetric equations may have asymmetric solutions. And also there are various symmetry breaking in the world[2, 3]. Dynamical symmetry breaking of extended gauge symmetries is given[4]. The Ref.[5] shows dynamical symmetry breaking in the sea of the nucleon, and the Ref.[6] investigates dynamical electroweak symmetry breaking by a neutrino condensate.

Spontaneous symmetry breaking plays an important role for constructing the different unified theories of the electroweak and the strong interactions, as well as gravity theory[7]. But the fundamental scalar field, e.g. Higgs particle, has not been discovered up to now, even though the low energy limit of finding Higgs particle has been raised to very high[8], especially in testing the standard model of the weak-electromagnetic interactions. The different grand unified theories have many parameters adjusted to fit the experiments, which make the theoretical predication to physical properties be decreased. On the other hand, there are other mechanisms generating particle's masses [9-12]. The Ref.[13] indicates that if the vacuum polarization tensor has a pole at light-like momenta, gauge field may acquire mass. A classical σ model of chiral symmetry breaking was given in Ref. [14], and an in-medium QMC model parameterization quark condensation in nuclear matter etc are studied in Ref[15, 16].

The pure interactions mediated by swapped mesons between fermions and antifermions possibly yield vacuum condensation of the fermion-antifermions pair [17], which makes vacuum degeneracy appears. Ref. [18] researched spontaneous and dynamical breaking of mean field symmetries in the proton-neutron quasiparticle random phase approximation and the description of double beta decay transitions. And dynamical chiral symmetry breaking in gauge theories with extra dimensions is also well described[19].

Dynamical electroweak breaking and latticized extra dimensions are shown up[20], using dynamical breaking, one may make fermions and bosons get masses, and may make the free adjusted parameters decrease, even to a dynamical group of one parameter.

When considering the physical effect of a system coming from another system, a general quantitative causal conservation principle must be satisfied[21]. Using the homeomorphic map transformation satisfying the general quantitative causal conservation principle, Ref.[22] solves the hard problem of the non-perfect properties of the Volterra process, the topological current invariants in Riemann-Cartan manifold and spacetime defects still satisfy the general quantitative causal conservation principle[23]. This paper illustrates the fact that σ and π^0 without electric charges having electromagnetic interaction effects coming from their inner constructions are just result satisfying the general causal conservation rule, i.e., the general quantitative causal conservation principle is essential for researching consistency of the model.

In general analyzing vacuum degeneracy, one studies only the degeneracy vacuum state originated from the self-action of scalar fields, one usually neglects the vacuum degeneracy originated from the interactions of different fields.

In this paper, Sect.2 gives the basic formulation; Sect.3 studies different condensation about fermions and antifermions; Sect.4 gives the concrete expressions of different mass spectrum about different vacuum breaking and dynamical breaking, and shows that the general four dimensional relations between different currents of the nuclear matter systems with J$\neq$ 0 and those with $J = 0$; the last Sect. is summary and conclusion.

2 Basic Formulation

The Lagrangian of the general σ-model with the symmetries of chiral $SU(2)_L \times SU(2)_R$ and electromagnetic $U(1)_{EM}$ is

$$\mathfrak{L}_j = \mathfrak{L} + \overline{\eta}\psi + \overline{\psi}\eta + J_\sigma \sigma + \mathbf{J}_\pi \cdot \pi + J_{A_\mu} A_\mu. \tag{2.2}$$

Euler-Lagrange Equations of the system are

$$[\gamma^\mu \partial_\mu - ieA_\mu + g(\sigma(x) + i\tau \cdot \pi(x)\gamma_5)]\psi(x) - \eta(x) = 0, \tag{2.3}$$

$$\overline{\psi}(x)[-\gamma^\mu \overleftarrow{\partial_\mu} - ie\gamma^\mu A_\mu + g(\sigma(x) + i\tau \cdot \pi(x)\gamma_5)] - \overline{\eta}(x) = 0, \tag{2.4}$$

$$(\Box + \lambda\nu^2)\sigma(x) - g\overline{\psi}(x)\psi(x) - \lambda\sigma(x)(\sigma^2(x) + \pi^2(x)) + J_\sigma(x) = 0, \tag{2.5}$$

$$(\Box + \lambda\nu^2)\pi(x) - e^2 A_\mu^2(x)\pi(x) - g\overline{\psi}(x)i\tau\gamma_5\psi(x) - \lambda\pi(x)(\sigma^2(x) + \pi^2(x)) + \mathbf{J}_\pi(x) = 0, \tag{2.6}$$

and

$$\partial_\nu F^{\mu\nu} + ie\overline{\psi}(x)\gamma^\mu\psi(x) - e^2 A_\mu(x)\pi^2(x) + J_{A_\mu} = 0. \tag{2.7}$$

Then we have

$$\langle\overline{\psi}(x)\gamma^\mu\partial_\mu\psi(x)\rangle_0^J - ie\langle\overline{\psi}(x)A_\mu(x)\psi(x)\rangle_0^J + g\langle\overline{\psi}(x)\sigma(x)\psi(x)\rangle_0^J + i\langle\overline{\psi}(x)\tau\cdot\pi(x)\gamma_5\psi(x)\rangle_0^J - \langle\overline{\psi}(x)\rangle_0^J\eta(x) = 0 \tag{2.8}$$

and

$$\langle\overline{\psi}(x)\gamma^\mu \overleftarrow{\partial\mu} \psi(x)\rangle_0^J + ie\langle\overline{\psi}(x)A_\mu(x)\psi(x)\rangle_0^J - g\langle\overline{\psi}(x)\sigma(x)\psi(x)\rangle_0^J - i\langle\overline{\psi}(x)\tau\cdot\pi(x)\gamma_5\psi(x)\rangle_0^J + \overline{\eta}(x)\langle\psi(x)\rangle_0^J = 0. \tag{2.9}$$

We can further obtain

$$(\Box + \lambda\nu^2)\langle\sigma(x)\rangle_0^J - g\langle\overline{\psi}(x)\psi(x)\rangle_0^J - \lambda\langle\sigma(x)(\sigma^2(x) + \pi^2(x)\rangle_0^J + J_\sigma(x) = 0, \tag{2.10}$$

$$(\Box + \lambda\nu^2)\langle\pi(x)\rangle_0^J - e^2\langle A_\mu^2(x)\pi(x)\rangle_0^J - g\langle\overline{\psi}(x)i\tau\gamma_5\psi(x)\rangle_0^J - \lambda\langle\pi(x)(\sigma^2(x) + \pi^2(x))\rangle_0^J + \mathbf{J}_\pi(x) = 0, \tag{2.11}$$

$$\langle\partial_\nu F^{\mu\nu}\rangle_0^J + ie\langle\overline{\psi}(x)\gamma^\mu\psi(x)\rangle_0^J - e^2\langle A_\mu(x)\pi^2(x)\rangle_0^J + J_{A_\mu}(x) = 0, \tag{2.12}$$

in which for any field, we can define $\langle Y(x)\rangle_0^J \equiv \langle 0_{out}|Y(x)|0_{in}\rangle_0^J / \langle 0_{out}|0_{in}\rangle_0^J$.

The generating functional of the system is

$$Z(J) \equiv \int \left[D\overline{\psi}\right][D\psi][D\sigma][D\pi][DA_\mu]\exp\left(\frac{i}{\hbar}\int d^4x\mathfrak{L}_J\right) \tag{2.13}$$

Using the generating function one have

$$\langle Y(x)\rangle_0^J = \hbar\frac{\delta W}{\delta J_Y(x)}, \tag{2.14}$$

where $Z = e^{iW}$.

On the other hand, using the method of deducing connection Green function from Green function in quantum field theory[24, 25] we can have

$$\langle\sigma^3(x)\rangle_0^J = (\langle\sigma(x)\rangle_0^J)^3 + 3\frac{\hbar}{i}\langle\sigma(x)\rangle_0^J\frac{\delta\langle\sigma(x)\rangle_0^J}{\delta J_\sigma(x)} + (\frac{\hbar}{i})^2\frac{\delta^2\langle\sigma(x)\rangle_0^J}{\delta J_\sigma(x)\delta J_\sigma(x)} + \cdots, \tag{2.15}$$

$$\begin{aligned}\langle\sigma(x)\pi^2(x)\rangle_0^J = (\langle\pi^2(x)\rangle_0^J)^2\langle\sigma(x)\rangle_0^J + \frac{\hbar}{i}\langle\sigma(x)\rangle_0^J\frac{\delta\langle\pi(x)\rangle_0^J}{\delta\mathbf{J}_\pi(x)}\\ + 2\frac{\hbar}{i}\langle\pi(x)\rangle_0^J\cdot\frac{\delta\langle\sigma(x)\rangle_0^J}{\delta\mathbf{J}_\pi(x)} + (\frac{\hbar}{i})^2\frac{\delta^2\langle\sigma(x)\rangle_0^J}{\delta\mathbf{J}_\pi(x)\cdot\delta\mathbf{J}_\pi(x)} + \cdots,\end{aligned} \tag{2.16}$$

$$\begin{aligned}\langle\pi(x)A_\mu^2(x)\rangle_0^J = \quad & (\langle A_\mu(x)\rangle_0^J)^2\langle\pi(x)\rangle_0^J + \frac{\hbar}{i}\langle\pi(x)\rangle_0^J\frac{\delta\langle A_\mu(x)\rangle_0^J}{\delta J_{A_\mu}(x)}\\ & +2\frac{\hbar}{i}\langle A_\mu(x)\rangle_0^J\frac{\delta\langle\pi(x)\rangle_0^J}{\delta J_{A_\mu}(x)} + (\frac{\hbar}{i})^2\frac{\delta^2\langle\pi(x)\rangle_0^J}{\delta J_{A_\mu}(x)\cdot\delta J_{A_\mu}(x)} + \cdots,\end{aligned} \tag{2.17}$$

$$\begin{aligned}\langle A_\mu(x)\pi^2(x)\rangle_0^J = \quad & \langle A_\mu(x)\rangle_0^J(\langle\pi(x)\rangle_0^J)^2 + \frac{\hbar}{i}\langle A_\mu(x)\rangle_0^J\frac{\delta\langle\pi(x)\rangle_0^J}{\delta\mathbf{J}_\pi(x)}\\ & +2\frac{\hbar}{i}\langle\pi(x)\rangle_0^J\cdot\frac{\delta\langle A_\mu(x)\rangle_0^J}{\delta\mathbf{J}_\pi(x)} + (\frac{\hbar}{i})^2\frac{\delta^2\langle A_\mu(x)\rangle_0^J}{\delta\mathbf{J}_\pi(x)\cdot\delta\mathbf{J}_\pi(x)} + \cdots,\end{aligned} \tag{2.18}$$

which are just a kind of new power expansion about the little quantity $\hbar$, which is essential for researching the physics of different power series about $\hbar$. Because there are possible condensations of $\langle\overline{\psi}(x)\psi(x)\rangle_0^J$, $\langle\overline{\psi}(x)i\tau\gamma_5\psi(x)\rangle_0^J$ and $\langle\overline{\psi}(x)\gamma_\mu\psi(x)\rangle_0^J$ in Eqs. (2.10-12), respectively, we have

$$\langle\overline{\psi}(x)\sigma(x)\psi(x)\rangle_0^J = \langle\sigma(x)\rangle_0^J\langle\overline{\psi}(x)\psi(x)\rangle_0^J + \frac{\hbar}{i}\frac{\delta\langle\overline{\psi}(x)\psi(x)\rangle_0^J}{\delta J_\sigma(x)} + \cdots, \tag{2.19}$$

$$\langle\overline{\psi}(x)i\tau\cdot\pi(x)\gamma_5\psi(x)\rangle_0^J = \langle\pi(x)\rangle_0^J\cdot\langle\overline{\psi}(x)i\tau\gamma_5\psi(x)\rangle_0^J + \frac{\hbar}{i}\frac{\delta\langle\overline{\psi}(x)i\tau\gamma_5\psi(x)\rangle_0^J}{\delta\mathbf{J}_\pi(x)} + \cdots, \tag{2.20}$$

$$\langle\overline{\psi}(x)A_\mu(x)\gamma_\mu\psi(x)\rangle_0^J = \langle A_\mu(x)\rangle_0^J\langle\overline{\psi}(x)\gamma_\mu\psi(x)\rangle_0^J + \frac{\hbar}{i}\frac{\delta\langle\overline{\psi}(x)\gamma_\mu\psi(x)\rangle_0^J}{\delta J_{A_\mu}(x)} + \cdots. \tag{2.21}$$

Hence, we obtain

$$\begin{aligned}&\langle\overline{\psi}(x)\gamma^\mu\partial\mu\psi(x)\rangle_0^J - ie\langle A_\mu(x)\rangle_0^J\langle\overline{\psi}(x)\gamma^\mu\psi(x)\rangle_0^J + g\langle\sigma(x)\rangle_0^J\langle\overline{\psi}(x)\psi(x)\rangle_0^J + ig\langle\pi(x)\rangle_0^J\langle\overline{\psi}(x)\tau\gamma_5\psi(x)\rangle_0^J\\ &- \langle\overline{\psi}(x)\rangle_0^J\eta(x) - e\hbar\frac{\delta\langle\overline{\psi}(x)\psi(x)\rangle_0^J}{\delta J_{A_\mu}(x)} + g\frac{\hbar}{i}\frac{\delta\langle\overline{\psi}(x)\psi(x)\rangle_0^J}{\delta J_\sigma(x)} + g\hbar\frac{\delta\langle\overline{\psi}(x)\psi(x)\rangle_0^J}{\delta\mathbf{J}_\pi(x)} + \cdots = 0,\end{aligned} \tag{2.22}$$

$$-\langle\overline{\psi}(x)\gamma^{\mu}\overleftarrow{\partial_{\mu}}\psi(x)\rangle_0^J - ie\langle A_{\mu}(x)\rangle_0^J\langle\overline{\psi}(x)\gamma^{\mu}\psi(x)\rangle_0^J + g\langle\sigma(x)\rangle_0^J\langle\overline{\psi}(x)\psi(x)\rangle_0^J + ig\langle\pi(x)\rangle_0^J\langle\overline{\psi}(x)\tau\gamma_5\psi(x)\rangle_0^J$$

$$-\bar{\eta}(x)\langle\psi(x)\rangle_0^J - e\hbar\frac{\delta\langle\overline{\psi}(x)\psi(x)\rangle_0^J}{\delta J_{A_{\mu}}(x)} + g\frac{\hbar}{i}\frac{\delta\langle\overline{\psi}(x)\psi(x)\rangle_0^J}{\delta J_{\sigma}(x)} + g\hbar\frac{\delta\langle\overline{\psi}(x)\psi(x)\rangle_0^J}{\delta \mathbf{J}_{\pi}(x)} + \cdots = 0, \tag{2.23}$$

and we can have

$$(\Box + \lambda\nu^2)\langle\sigma(x)\rangle_0^J = g\langle\overline{\psi}(x)\psi(x)\rangle_0^J + \lambda\langle\sigma(x)\rangle_0^J[(\langle\sigma(x)\rangle_0^J)^2 + (\langle\pi(x)\rangle_0^J)^2] + \lambda\frac{\hbar}{i}[3\langle\sigma(x)\rangle_0^J\frac{\delta\langle\sigma(x)\rangle_0^J}{\delta J_{\sigma}(x)} +$$

$$\langle\sigma(x)\rangle_0^J\frac{\delta\langle\pi(x)\rangle_0^J}{\delta\mathbf{J}_{\pi}(x)} + 2\langle\pi(x)\rangle_0^J\cdot\frac{\delta\langle\sigma(x)\rangle_0^J}{\delta\mathbf{J}_{\pi}(x)}] + \lambda(\frac{\hbar}{i})^2[\frac{\delta^2\langle\sigma(x)\rangle_0^J}{\delta J_{\sigma}(x)\delta J_{\sigma}(x)} + \frac{\delta^2\langle\sigma(x)\rangle_0^J}{\delta\mathbf{J}_{\pi}(x)\cdot\delta\mathbf{J}_{\pi}(x)}] - J_{\sigma}(x) + \cdots, \tag{2.24}$$

$$(\Box + \lambda\nu^2)\langle\pi(x)\rangle_0^J = g\langle\overline{\psi}(x)i\tau\gamma_5\psi(x)\rangle_0^J + \lambda\langle\pi(x)\rangle_0^J[(\langle\sigma(x)\rangle_0^J)^2 + (\langle\pi(x)\rangle_0^J)^2] + \lambda\frac{\hbar}{i}[3\langle\pi(x)\rangle_0^J\frac{\delta\langle\pi(x)\rangle_0^J}{\delta\mathbf{J}_{\pi}(x)}$$

$$+2\langle\sigma(x)\rangle_0^J\frac{\delta\langle\pi(x)\rangle_0^J}{\delta J_{\sigma}(x)} + \langle\pi(x)\rangle_0^J\cdot\frac{\delta\langle\sigma(x)\rangle_0^J}{\delta J_{\sigma}(x)}] + \lambda(\frac{\hbar}{i})^2[\frac{\delta^2\langle\pi(x)\rangle_0^J}{\delta J_{\sigma}(x)\cdot\delta J_{\sigma}(x)} + \frac{\delta^2\langle\pi(x)\rangle_0^J}{\delta\mathbf{J}_{\pi}(x)\cdot\delta\mathbf{J}_{\pi}(x)}] - \mathbf{J}_{\pi}(x) +$$

$$e^2[(\langle A_{\mu}(x)\rangle_0^J)^2\langle\pi(x)\rangle_0^J + \frac{\hbar}{i}\langle\pi(x)\rangle_0^J\frac{\delta\langle A_{\mu}(x)\rangle_0^J}{\delta J_{A_{\mu}}(x)} + 2\frac{\hbar}{i}\langle A_{\mu}(x)\rangle_0^J\frac{\delta\langle\pi(x)\rangle_0^J}{\delta J_{A_{\mu}}(x)} + (\frac{\hbar}{i})^2\frac{\delta^2\langle\pi(x)\rangle_0^J}{\delta J_{A_{\mu}}(x)\delta J_{A_{\mu}}(x)}] + \cdots, \tag{2.25}$$

$$\begin{aligned}\langle\partial_{\nu}F^{\mu\nu}\rangle_0^J + ie\langle\overline{\psi}(x)\gamma^{\mu}\psi(x)\rangle_0^J - e^2[(\langle\pi(x)\rangle_0^J)^2\langle A_{\mu}(x)\rangle_0^J + \frac{\hbar}{i}\langle A_{\mu}(x)\rangle_0^J\frac{\delta\langle\pi(x)\rangle_0^J}{\delta\mathbf{J}_{\pi}(x)} \\ +2\frac{\hbar}{i}\langle\pi(x)\rangle_0^J\cdot\frac{\delta\langle A_{\mu}(x)\rangle_0^J}{\delta\mathbf{J}_{\pi}(x)} + (\frac{\hbar}{i})^2\frac{\delta^2\langle\pi(x)\rangle_0^J}{\delta\mathbf{J}_{\pi}(x)\cdot\delta\mathbf{J}_{\pi}(x)} + \cdots] + J_{A_{\mu}}(x).\end{aligned} \tag{2.26}$$

And we can further obtain

$$\langle\partial_{\mu}(\overline{\psi}(x)\gamma^{\mu}\psi(x))\rangle_0^J = \langle\overline{\psi}(x)\rangle_0^J\eta(x) - \overline{\eta}(x)\langle\psi(x)\rangle_0^J, \tag{2.27}$$

when $\overline{\eta} = \eta = 0$, it follows that

$$\partial_{\mu}\overline{(\psi}(x)\gamma^{\mu}\psi(x)) = 0, \; i.e., \; \partial_{\mu}j^{\mu} = 0. \tag{2.28}$$

We neglect the powers with $\hbar$ in the power series, and take external sources into zero, therefore, we deduce

$$g\langle\overline{\psi}(x)\psi(x)\rangle_0^J\mid_{J=0} + \lambda\sigma_0(\sigma_0^2 + \pi_0^2 - \nu^2) = 0, \tag{2.29}$$

$$ig\langle\overline{\psi}(x)\gamma_5\tau\psi(x)\rangle_0^J\mid_{J=0} + \lambda\pi_0(\sigma_0^2 + \pi_0^2 - \nu^2) = 0, \tag{2.30}$$

$$\langle\partial_\nu F^{\mu\nu}\rangle_0^J \mid_{J=0} + ie\langle(\overline{\psi}(x)\gamma^\mu\psi(x)\rangle_0^J \mid_{J=0} = 0, \tag{2.31}$$

where $\sigma_0 = \langle\sigma(x)\rangle_0^J \mid_{J=\,0}$and $\pi_0 = \langle\pi(x)\rangle_0^J \mid_{J=\,0}$.

Analogous to Ref.[26]$'$s research, fermion$'$s propagator is

$$\langle\overline{\psi}(x)\psi(x')\rangle_0^J = \frac{1}{(2\pi)^4}\int^\Lambda \frac{-e^{i(x-x')\,\cdot\,p}d^4p}{\gamma^\mu\cdot p_\mu - ig\langle\sigma(x)\rangle_0^J + g\tau\cdot\langle\pi(x)\rangle_0^J\gamma_5 - e\gamma^\mu\langle A_\mu(x)\rangle_0^J}, \tag{2.32}$$

where Λ is the cutting parameter, Eqs.(2.28-32) are the basic equations relative to both dynamical breaking and vacuum breaking.

3 Different Condensations About Fermions and Antifermions and the Four Dimensional General Different Currents

We now generally investigate the different condensations about fermions and antifermions.

When $\sigma_0 \neq 0$, $\langle\overline{\psi}(x)\psi(x)\rangle_0^J \mid_{J=0} \neq 0$, we evidently have

$$\frac{ig\langle\overline{\psi}(x)\gamma_5\tau\psi(x)\rangle_0^J \mid_{J=0}}{g\langle\overline{\psi}(x)\psi(x)\rangle_0^J \mid_{J=0}} = \frac{\pi_0}{\sigma_0}, \tag{3.1}$$

then we generally have

$$\sigma_0 = Kg\langle\overline{\psi}(x)\psi(x)\rangle_0^J \mid_{J=0}, \tag{3.2}$$

$$\pi_0 = iKg\langle\overline{\psi}(x)\gamma_5\tau\psi(x)\rangle_0^J \mid_{J=0}, \tag{3.3}$$

where K is the parameter determined by physical experiments or theoretical model. Eq.(3.2) and (3.3) mean that σ_0 and π_0 are directly originated from the dynamical condensations of fermion-antifermion. The condensations also depend on K, which is different from the condensation mechanism before.

Analogous to Ref.[27], it shows that under some conditions the fundamental scalar fields are equivalent to the composed scalar fields.

Furthermore, we have

$$ic\langle\rho_e(x)\rangle_0^J \mid_{J=0} = \langle\partial_\nu F^{4\nu}(x)\rangle_0^J \mid_{J=0} = -ie\langle\psi^+(x)\psi(x)\rangle_0^J \mid_{J=0}, \tag{3.4}$$

$$\langle j_e^i\rangle_0^J \mid_{J=0} = \langle\partial_\nu F^{i\nu}(x)\rangle_0^J \mid_{J=0} = -ie\langle\overline{\psi}(x)\gamma^i\psi(x)\rangle_0^J \mid_{J=0}, \tag{3.5}$$

where ρ_e and j_e^i are the electric charge density and the electric current density, respectively, in nuclear matter. We also may discuss the current by means of Ref.[23]'s analogous method. Therefore, we obtain the average relation of nuclear matter density and electric charge density at the situation without external source as follows

$$\rho_g \equiv \langle\rho_B(x)\rangle_0^J \mid_{J=0} = \frac{-c}{e}\langle\rho_e(x)\rangle_0^J \mid_{J=0}, \tag{3.6}$$

where ρ_g is the ground state density of the fermi doublet, and $\rho_B(x) = \psi^+(x)\psi(x)$ is the density operator of the proton and neutron isospin doublet, Eq.(3.6)'s physical meaning is that the ground state of nucleon density equates to the condensation of the electric charge density divided by electronic charge and multiplied by $-c$, the condensation is the distribution of the ground state density of charged particles in nucleons.

We further get

$$\frac{i}{e}\langle j_e^i\rangle_0^J \mid_{J=0}= \frac{i}{e}\langle \partial_\nu F^{i\nu}(x)\rangle_0^J \mid_{J=0}= \langle j^i\rangle_0^J \mid_{J=0}\equiv j_0^i, \tag{3.7}$$

where $j^i = \overline{\psi}(x)\gamma^i\psi(x)$ is a vector current density of the nuclear matter.

On the other hand, because the interactions of $U_{EM}(1)$ and $SU_C(3)$ gauge fields generally affect the state of the matter, when the corresponding external sources $J_{gauge} = (J_{A_\mu}, J_{A_\mu^\kappa}) \neq 0$ (J_{A_μ} and $J_{A_\mu^\kappa}$ are external sources of the interactions of $U_{EM}(1)$ and $SU_C(3)$ gauge fields, respectively, κ is $SU_C(3)$ color gauge group index), we may generally assume a general equivalent velocity $\mathbf{v}$ (of the nuclear matter system with $J_{gauge} \neq 0$) relative to the primordial (or called, ground state′s) nuclear matter system with $J_{gauge} = 0$, because the equivalent relative velocity $\mathbf{v}$ is originated from the external sources $J_{gauge} = (J_{A_\mu}, J_{A_\mu^\alpha})$ wth Lorentz subscriptions. In fact, the actions of the external sources make the nuclear matter system with $J_{gauge} \neq 0$ have the excited equivalently relative velocity $\mathbf{v}$. Therefore, the velocity $\mathbf{v}$ is the function of the external sources, i. e., $\mathbf{v} = \mathbf{v}(J_{A_\mu}, J_{A_\mu^\kappa}) = \mathbf{v}(J_{gauge})$. Using a general Lorentz transformation we can obtain the relations of the four dimensional general current of nuclear matter system (with $J_{gauge} \neq 0$) relative to the nuclear matter system (with $J_{gauge}= 0$) of the ground state as follows

$$\mathbf{j}' = \mathbf{j}_0 + \mathbf{v}(J_{gauge})\left[(\frac{1}{\sqrt{1-\frac{\mathbf{v}^2(J_{gauge})}{c^2}}} - 1)\frac{\mathbf{j}_0\cdot\mathbf{v}(J_{gauge})}{c^2} - \frac{\rho_g}{\sqrt{1-\frac{\mathbf{v}^2(J_{gauge})}{c^2}}}\right], \tag{3.8}$$

$$\rho' = \frac{\rho_g - \frac{\mathbf{j}_0\,\cdot\,\mathbf{v}\,(J_{gauge})}{c^2}}{\sqrt{1-\frac{\mathbf{v}^2(J_{gauge})}{c^2}}}. \tag{3.9}$$

We, thus, can generally assume the velocity $\mathbf{v}(J_{gauge})$ linearly depends on the external sources. Therefore, we can obtain a general expression

$$\mathbf{v}(J_{gauge}) = \alpha_{A_\mu} J_{A_\mu} + \alpha_{A_\mu^\kappa} J_{A_\mu^\kappa} \tag{3.10}$$

in which α_{A_μ} and $\alpha_{A_\mu^\kappa}$ are the corresponding relative coupling constants of external sources J_{A_μ} and $J_{A_\mu^\kappa}$, respectively. Thus, Eqs.(3.8) and (3.9) may be rewritten as two general expressions linearly depending on the external sources as follows

$$\mathbf{j}' = \mathbf{j}_0 + (\alpha_{A_\mu} J_{A_\mu} + \alpha_{A_\mu^\kappa} J_{A_\mu^\kappa})\{(\frac{1}{\sqrt{1-\frac{(\alpha_{A_\mu} J_{A_\mu} + \alpha_{A_\mu^\kappa} J_{A_\mu^\kappa})^2}{c^2}}} \tag{3.11}$$

$$-1)\frac{\mathbf{j}_0\cdot(\alpha_{A_\mu} J_{A_\mu} + \alpha_{A_\mu^\kappa} J_{A_\mu^\kappa})}{c^2} - \frac{\rho_g}{\sqrt{1-\frac{(\alpha_{A_\mu} J_{A_\mu} + \alpha_{A_\mu^\kappa} J_{A_\mu^\kappa})^2}{c^2}}}\},$$

$$\rho' = \frac{\rho_g - \frac{\mathbf{j}_0 \cdot (\alpha_{A_\mu} J_{A_\mu} + \alpha_{A_\mu^\kappa} J_{A_\mu^\kappa})}{c^2}}{\sqrt{1 - \frac{(\alpha_{A_\mu} J_{A_\mu} + \alpha_{A_\mu^\kappa} J_{A_\mu^\kappa})^2}{c^2}}}. \tag{3.12}$$

and the consistent condition is

$$\left| \alpha_{A_\mu} J_{A_\mu} + \alpha_{A_\mu^\kappa} J_{A_\mu^\kappa} \right| < c \tag{3.13}$$

In order to make the theory concrete, we consider a case when the external source $J_{A_\mu^\kappa}$ equates to zero but external J_{A_μ}. Then we gain the general case that there exists electromagnetic field, Eqs.(3.11) and (3.12), thus, can be represented as

$$\mathbf{j}' = \mathbf{j}_0 + \alpha_{A_\mu} J_{A_\mu} \{ (\frac{1}{\sqrt{1 - \frac{(\alpha_{A_\mu} J_{A_\mu})^2}{c^2}}} - 1) \frac{\mathbf{j}_0 \cdot \alpha_{A_\mu} J_{A_\mu}}{c^2} - \frac{\rho_g}{\sqrt{1 - \frac{(\alpha_{A_\mu} J_{A_\mu})^2}{c^2}}} \}, \tag{3.14}$$

$$\rho' = \frac{\rho_g - \frac{\mathbf{j}_0 \cdot \alpha_{A_\mu} J_{A_\mu}}{c^2}}{\sqrt{1 - \frac{(\alpha_{A_\mu} J_{A_\mu})^2}{c^2}}}, \tag{3.15}$$

and the corresponding consistent condition is $\left| \alpha_{A_\mu} \right| < \frac{c}{J_{A_\mu}}$. When α_{A_μ} is generally chosen as the motion direction $\mathbf{e}_x$, and J_{A_μ} is taken as magnetic field B, we, thus, can have

$$j'_x = \frac{j_{0x} - \rho_g \alpha B}{\sqrt{1 - \frac{(\alpha B)^2}{c^2}}}, \tag{3.16}$$

$$\rho' = \frac{\rho_g - \frac{\alpha B}{c^2} j_{0x}}{\sqrt{1 - \frac{(\alpha B)^2}{c^2}}}, \tag{3.17}$$

where α is the small parameter determined by the nuclear physical experiments under the external magnetic field B.

In order to test the theory, considering the case of $j_{0x} = 0$, in Eq.(3.17) we have

$$\rho' = \frac{\rho_g}{\sqrt{1 - \frac{(\alpha B)^2}{c^2}}}. \tag{3.18}$$

Because α is the coupling parameter, Eq(3.18) shows the relation of density ρ's coupling effect with external magnetic field, which conforms to Ref.[28]$'$s research about dense nuclear matter in a strong magnetic field.

4 Different Mass Spectrum about Different Dynamical Breaking and Vacuum Breaking

Because σ_0 and π_0 may be made from the condensations of fermion-antifermion, we can discuss the concrete expressions of different mass spectrum about different dynamical breaking and different spontaneous vacuum symmetry breaking as follows:

(i) When considering the following dynamical breaking

$$\langle\overline{\psi}(x)\psi(x)\rangle_0^J \mid_{J=0}\neq 0,\ \langle\overline{\psi}(x)\gamma_5\tau\psi(x)\rangle_0^J \mid_{J=0}= 0, \tag{4.1}$$

we has

$$\pi_0 = 0,\ \lambda\sigma_0(\nu^2-\sigma_0^2) = g\langle\overline{\psi}(x)\psi(x)\rangle_0^J \mid_{J=0}= -gtrS_F(0), \tag{4.2}$$

the corresponding spontaneous vacuum symmetry breaking is

$$\sigma(x) \longrightarrow \sigma(x)+\sigma_0, \tag{4.3}$$

the Lagrangian density, thus, is

$$\begin{aligned}\mathcal{L} = \quad & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \overline{\psi}(x)[\gamma^\mu(\partial_\mu - ieA_\mu)+m_f]\psi(x) - g\overline{\psi}(x)[\sigma(x)+i\tau\cdot\pi(x)\gamma_5]\psi(x)] \\ & -\frac{1}{2}(\partial_\mu\sigma(x))^2 - \frac{1}{2}m_\sigma^2\sigma^2(x) - \frac{1}{2}(\partial_\mu + ieA_\mu)\pi^+(x)\cdot(\partial_\mu - ieA_\mu)\pi(x) - \frac{1}{2}m_\pi^2\pi^2(x) \\ & -\frac{\lambda}{4}(\sigma^2(x)+\ \pi(x))^2 - \lambda\sigma_0\sigma(x)(\sigma^2(x)+\pi^2(x)) - gtrS_F(0)\sigma(x).\end{aligned} \tag{4.4}$$

One obtains that the fermion doublet masses are

$$m_f = g\sigma_0. \tag{4.5}$$

Masses of $\sigma(x)$ and $\pi(x)$, respectively, are

$$m_\sigma^2 = \lambda(3\sigma_0^2 - \nu^2) = 2\lambda\sigma_0^2 + gS_F(0)/\sigma_0, \tag{4.6}$$

$$m_\pi^2 = \lambda(\sigma_0^2 - \nu^2) = gtrS_F(0)/\sigma_0. \tag{4.7}$$

Thus, when there is no dynamical breaking, we obtain $\sigma_0^2 = \nu^2$, which just shows ν^2's physical meaning, i.e., σ_0 is, in this case, just the spontaneous vacuum breaking parameter, and $m_\pi^2 = 0$. Even so, σ particles and fermions acquire masses, namely $m_\sigma^2 = 2\nu^2$, $m_f = g\,|\nu|$. Therefore, the masses of σ particle and fermion doublet naturally come from only the vacuum breaking structure. In general case, when there exist both dynamical breaking and spontaneous vacuum breaking, not only π meson and fermions gain masses, but also σ and π masses are not equal. More generally, we may take $\sigma_0' = \langle\overline{\psi}(x)\psi(x)\rangle_0^J$ in which σ_0' is the running spontaneous vacuum breaking value. It means that σ_0' is the exciting state, which make fermion doublet, σ particle and π gain effective masses relative to different external sources.

(ii) when $\sigma_0 = 0$, $\pi_0 = 0$, analogous to the research about Eqs.(4.6) and (4.7), we get $\sigma(x)$ and $\pi(x)$ meson having the same mass[24]

$$m_\sigma^2 = m_\pi^2 = -\lambda\nu^2. \tag{4.8}$$

Further using Eq.(4.5) in the cases of $\sigma_0 = 0$ and $\pi_0 = 0$, we obtain the fermion doublet keeping no mass.

(iii) General dynamical breaking

We now consider a general dynamical breaking. From Eqs.(3.2) and (3.3) we see that

$$\sigma_0 = Kg\langle\overline{\psi}(x)\psi(x)\rangle_0^J \mid_{J=0}\neq 0,\ \ \pi_0 = iKg\langle\overline{\psi}(x)\gamma_5\tau\psi(x)\rangle_0^J \mid_{J=0}\neq 0. \tag{4.9}$$

Then the corresponding spontaneous vacuum symmetry breaking are

$$\sigma(x) \longrightarrow \sigma(x) + \sigma_0,\ \ \pi(x) \longrightarrow \pi(x) + \varepsilon\pi_0,\ 0 \leq \varepsilon \leq 1, \tag{4.10}$$

where ε is a running breaking coupling parameter determined by different physical experiments.

Because electromagnetic interaction is very weaker than strong interaction, electromagnetic interaction may be neglected. The corresponding Lagrangian, is

$$\mathcal{L} = -\overline{\psi}(x)[\gamma^\mu\partial_\mu + m_f]\psi(x) - g\overline{\psi}(x)[\sigma(x) + i\tau\cdot\pi(x)\gamma_5]\psi(x) - \frac{1}{2}(\partial_\mu\sigma(x))^2 - \frac{m_\sigma^2}{2}\sigma^2(x) - \frac{1}{2}(\partial_\mu\pi(x))^2$$

$$-\frac{\lambda}{2}[(\sigma_0^2 + \varepsilon^2\pi_0^2 - \nu^2)\pi^2 + 2(\varepsilon\pi_0.\pi)^2] - \frac{\lambda}{4}(\sigma^2(x) +\ \pi^2(x))^2 - \lambda(\sigma_0\sigma(x) + \varepsilon\pi_0.\pi(x))(\sigma^2(x) + \pi^2(x))$$

$$- 2\lambda\sigma_0(\varepsilon\pi_0\cdot\pi(x))\sigma(x) - \frac{\lambda}{2}(\sigma_0^2 + \varepsilon^2\pi_0^2 - \nu^2)(\sigma_0\sigma(x) + \varepsilon\pi_0.\pi(x)) - \frac{\lambda}{4}(\sigma_0^2 + \varepsilon^2\pi_0^2 - \nu^2)^2, \tag{4.11}$$

where masses of the fermions and σ particle , respectively, are

$$m_N = g(\sigma_0 + i\varepsilon\tau\cdot\pi_0\gamma_5), \tag{4.12}$$

$$m_\sigma^2 = \lambda(3\sigma_0^2 + \varepsilon^2\pi_0^2 - \nu^2). \tag{4.13}$$

Because of

$$(\pi_0\cdot\pi)^2 = \pi_0^2\pi^2 + \sum_{\substack{i,j=1\\ i\neq j}}^{3}\left(\pi_{i0}\pi_{j0}\pi_i\pi_j - \pi_{i0}^2\pi_j^2\right) \tag{4.14}$$

Under the condition of $\sum_{\substack{i,j=1\\ i\neq j}}^{3}\pi_{i0}\pi_{j0}\pi_i\pi_j = \sum_{\substack{i,j=1\\ i\neq j}}^{3}\pi_{i0}^2\pi_j^2$, we obtain meson mass expression

$$m_\pi^2 = \lambda(\sigma_0^2 + 3\varepsilon^2\pi_0^2 - \nu^2). \tag{4.15}$$

When $\pi_0 = 0$ or $\varepsilon = 0$, the results (iii) are simplified into the results (i) above.

When there is pseudoscalar condensation $\langle\overline{\psi}(x)\tau\gamma_5\psi(x)\rangle_0^J \mid_{J=0}$, because the scalar condensation is stronger than the pseudoscalar condensation, the σ_0 is not equal to zero under existing pseudoscalar condensation.

From the above discussion, we may see what no needing Higgs particle, we naturally gain both fermion′s masses and boson′s (σ and π) masses. The mechanisms of gaining masses are more direct and useful for constructing the weak-electromagnetic standard model without Higgs fields. For making fermions and bosons in the other models acquire masses, it may make the too many adjusting parameters of fitting with the physical experiments in the usual unified models decrease. We, further, generally deduce that the masses of nucleons, σ and π have the effects coming from interactions with external source. It can be seen that σ and π^0 may be made from the different condensations of

fermion and antifermion. This lead to that σ and π^0 without electric charge have electromagnetic interaction effects coming from their inner construction. Using the all general research of this paper,we can very more study the interactions between different fundamental particles in general situation, all these will be written in the other papers.

5 Summary and Conclusion

We show up a general $SU(2)_L \times SU(2)_R \times U(1)_{EM}$ σ- model with external sources, dynamical breaking and spontaneous vacuum symmetry breaking. We present the general basic formulations of the model. This paper founds the different condensations about fermions and antifermions in which the concrete scalar and pseudoscalar condensed expressions of σ_0 and π_0 bosons are shown up. We have shown that σ and π^0 may be made from the different condensations of fermion and antifermion. We have discovered that σ and π^0 without electric charge have electromagnetic interaction effects coming from their inner construction, which is similar to neutron. Using a general Lorentz transformation and four dimensional condensed currents of the nuclear matter of the ground state with J = 0 we deduced the four dimensional general relations of different currents of the nuclear matter system with J $\neq$ 0 relative to the ground state$'$s nuclear matter system with J = 0, and give the relation of density ρ's coupling effect with external magnetic field. This conforms to Ref.[28]'s research about dense nuclear matter in a strong magnetic field. We also get the concrete expressions of different mass spectrum about different dynamical breaking and spontaneous vacuum breaking. This paper has given running spontaneous vacuum breaking value σ_0' in terns of the technique of external sources, has obtained spontaneous vacuum symmetry breaking based on the σ_0', which make nuclear fermion doublet, σ and π particles gain effective masses relative to external sources. We have found out the mechanisms of mass production of fermion doublet and bosons (σ and π). The mechanism is useful for constructing the unified weak-electromagnetic model without fundamental scalar fields. The effect of external sources and nonvanishing values of the scalar and pseudoscalar condensations are given in this theory, we generally deduce that the masses of nucleons, σ and π partly come from the interactions with different external sources.

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